

Toward a New Representation of Aristotle's Modal Logic

José Veríssimo Teixeira da Matta

March 1 2012

Abstract

The aim of this article is to rebuild an argument by Aristotle presented in *De Interpretatione* and its reconstruction in modern logical tools made by Łukasiewicz. As consequence of my rebuilding, I present a new symbolic representation of propositions concerning the modalities of necessity and possibility and I discuss the ontological status of such propositions.

Modal Logic has been the object of refined analysis by Jan Łukasiewicz in his classic book *Aristotle's Syllogistic*. There, after remembering us that, according to Aristotle, only propositions are necessary, impossible, possible, or contingent, he says¹ (concerning a proposition p) that:

“The proposition “ p ” is necessary, where “ p ” is the name of the proposition p , I shall use the expression: ‘It is necessary that p ’, where p is a proposition. So, for instance, instead of saying: ‘The proposition “man is animal” is necessary’, I shall say: ‘It is necessary that man should be an animal’. I shall express the other modalities in a similar way. Expressions like: ‘It is necessary that p ’, denoted here by Lp or ‘It is possible that p ’, denoted by Mp , I call *modal functions*; L and M , which respectively correspond to the words ‘it is necessary that’ and ‘it is possible that’, are *modal functors*, p is their *argument*. I say that L and M are proposition-forming functors of one propositional argument. Propositions beginning with L or their equivalents are called *apodeictic*, those beginning with M or their equivalents *problematic*. Non-modal propositions are called *assertoric*”

Aristotle's modal logic was translated by Łukasiewicz in chapter VI of the *Aristotle's Syllogistic* using modern logic symbolism, and as he tells us:

“This modern terminology will help us to give a clear exposition of Aristotle's propositional modal logic.”²

¹ŁUKASIEWICZ, 1955: P.134.

²IBIDEM

Two of the modal terms, *necessary* and *possible*, and their interrelations, are of fundamental importance. In the *De Interpretatione* Aristotle mistakenly asserts that possibility implies non-necessity, i.e., in our terminology:

(1) If it is possible that p , it is not necessary that p . He later sees that this cannot be right, because he accepts that necessity implies possibility, i.e.:

(2) If it is necessary that p , it is possible that p ,

and from (1) and (2) there would follow by the hypothetical syllogism that

(3) If it is necessary that p , it is not necessary that p , which is absurd.”

One question comes to mind after reading Lukasiewicz exposition: why Aristotle himself did not present the proposition as did Lukasiewicz, i.e., as it is represented in our modern times?

For example, remembering: ‘it is necessary that p ’, ‘it is possible that p ’.

Given that the representation of propositions by letters appear later in the *Analitic Prior*, the answer to our question is quite simple, the formulation with letters didn’t appear before for one simple reason: it was not invented yet.

In *Analitic Prior*, predicates and subjects, antecedent and consequent propositions, are represented with letters.

We must remark that the statement (3) above “If it is necessary that p , it is not necessary that p ” is named by both Aristotle and Lukasiewicz as *absurd*. Aristotle himself used the word *atopos*. We can translate *atopos* as something that can not occur; that can not be. There is here a limitation to explain a result which can not be admitted. Aristotle in at that moment did not support any conclusion like a explosion of contradictions as we will see later in *Metaphysics*³.

It is opportune to notice too that Aristotle not only discussed and discovered the modalities as logical functions, but he also tried to find connections between them using mechanisms of *conversion*, *inversion*, *deduction*, *implication* and *reduction*. And indeed, the great merit of Aristotle in his formulation of modal logic is his discovery of relations between different concepts of modalities and how to use and move between these modalities and how to put them in connection to each other, despite the fact irreducible modalities, *possible*, *necessary*, *impossible* and *contingent* are very different functors.

The most clear definition of the modality *possible* is the following:

“Something that can be or can not be, like the clothes which can be cut or can not be cut (19 a -10-15).”

I consider this presentation of *possible* very important, for whatever logical project about possibility.

³1007 b -20-25

So, despite of any force of Lukasiewicz's rewriting of the Aristotle's text the absurd obtained by those men, one in the antiquity and the other in the XX century, does not seem to correspond to the to what one may learn in *De Interpretatione*, if we carefully analyze the fatalist's argument in chapter IX of the book.

Indeed, in that argument there is a real error. Aristotle probably did this error forgetting for an unknown reason some of his previous writings or eventually also because he may have been very much economic in his writing, representing the modal statements with abbreviated formulae. He did not wrote the modal propositions integrally, and wrote statemental nuclei in incomplete way. For example (22 a 24 -27), he wrote:

“possible to be”;
“admissible to be”;
“not impossible to be”;
“not necessary to be”.

Unfortunately, some modal statements have proper formulae and nuances, which – if forgotten – conduct us almost necessarily, if not necessarily, toward errors. We will see below the nature of these errors.

The exposition of Lukasiewicz of the Aristotle's modal propositions follows the chapter XII and XIII of *De Interpretatione*. Nevertheless, chapter IX is partly very important to solve he question about the formal representation of modal statements in Aristotle's logical thought.

In chapter IX, the fatalist to make unsound the possibility of a third logical value presents the following argument⁴: if for the two contradictory statements (a) and (b) below the two values – True (*T*) and false (*F*) – are not sufficient:

(a) Tomorrow there will be a sea battle near Salamina Island,

“and”

(b) Tomorrow there won't be a sea battle near Salamina Island,

then, we will have for these two propositions a third value, which will be simultaneously the negation of both propositions (a) and (b), and this new value would not be true (*T*/1) nor false (*F*/0). Using this reasoning, the fatalist comes to the following proposition:

(4) “It is necessary that tomorrow neither will there be a sea battle near Salamina Island nor will there not be a sea battle near Salamina Island”.

The statement may if written: $L \sim(p \text{ and } \sim p)$, where p has values, 1 or 0, $\sim p$ values 0 or 1 and $L \sim(p \text{ and } \sim p)$ will be need to have a new value: 1/2 (neither one nor zero), or something like neither (a) nor (b) or 1/2. ⁵

⁴The exposition is merely schematic here.

⁵Ackrill 's translation of *De Interpretatione* in: 18 b-24-25 “Take a seabattle: it would have neither to happen nor not happen.”

That for some unknown reason this argument was taken as absurd (1/2 can not be there, because it makes no sense), the fatalist's experiment practically inaugurated what we know as modal logic.

Aristotle basically only corrected the fatalist concerning the use of the functor or connective *and* to express the necessity. Thus the error of the fatalist consists precisely in the articulation of the connective (or functor) *and* (neither... **nor** = **not** and **not**) with the modality *necessary*. On the other hand, the *possibility* (other basic modality) is articulated with the same two statements (p and $\sim p$), but with the functor *and*. Here, from Aristotle exposition in chapter IX, we come to the following equivalence as being a fundamental one in Aristotle's modal logic:

$$(5) \quad L(p \text{ or } \sim p) \equiv M(p \text{ and } \sim p).$$

Given that, contradictions are warranted, as we know from the semantic exposition⁶ in *De Interpretatione*. The contradiction of two propositions in the future is one exception to the rule of contradictory pair (RCP)⁷:

“Clearly then, it is not necessary that for every statement and its opposite negation one should be true and the other false. For what holds for things that are does not hold for things that are not but may possibly be or not be; with these it is as we have said.”

As we know, by the RCP, one of the propositions of a contradictory pair if the pair has singular propositions) should be true and the other false and this law concerns the hypothetical necessity; in fact hypothetical necessity means here past and present.

We must consider that the contradictions for sentences about the future don't risk to turn the system to triviality or inconsistency. Why?

The contradictions concerning the future appear in modal logic in the same way contradictions appear in the indifferent judgment thought by Vasil'ev⁸ They are *intrastatemental* contradictions and not *interstatemental* contradictions. They refer to the dictum and not to the modalities where the dictum is (applies).

That means according to our view that Aristotle discovered this crucial distinction which is fundamental for many no-classical logics: it is one thing to think the intrastatemental contradictions, it is another thing to think the interstatemental contradictions.

For example, consider an indifferent judgment in Vasil'ev:

$$A \text{ is } B \text{ and } \sim B,$$

and the modal sentence in Aristotle:

$$\text{It is possible } p \text{ and } \sim p.$$

⁶Ackrill 's translation of *De Interpretatione* in: 19 a 39- 19 b 4.

⁷RCP- about this rule, see Whitaker, C.W.A (Aristotle's *De Interpretatione* - Chapter- 9: The third exception to the RCP).

⁸Imaginary (non-aristotelian) logic, journal of the Ministry of Education, 1912.

In both cases the kind of contradictions does not produce any risk the logical system. The modal system, e.g., does not admit the following kind of contradiction:

$\sim M(p \text{ and } \sim p) \text{ and } M(p \text{ and } \sim p)$, which is in essence $\sim M \text{ and } M$.

In fact, the modal proposition is $M(p \text{ and } \sim p)$ and not $Mp \text{ or } M\sim p$, because p and $\sim p$ has the same ontological status and consequently can not be separated. In the limit, the cause of the individuation of sentences like S is P and S is not P , is the different ontological status that one has concerning the other. It doesn't happen with $M(p \text{ and } \sim p)$, because p and $\sim p$ form a unity, as they have the same ontological status. One proposition expresses a fact of just one ontological status, and no more than that.⁹

The abbreviated formulae for modal logic presented in the chapters XII and XIII of *De Interpretatione* must be read as the chapter IX tells us. Consequently, I propose to rebuild Aristotle's argument and Lukasiewicz formulation of that argument in the following way:

(6) (If it is possible that p and that $\text{non} - p$), it is not necessary that p and that $\text{non} - p$.

(7) If it is necessary that p or that $\text{non} - p$, (it is possibly that p and $\text{non} - p$).

From (6) and (7), cutting the middles represented by the brackets $(())$, in the hypothetic syllogism, we obtain the following result:

(8) If it is necessary that p or that $\text{non} - p$, it is not necessary that p and that $\text{non} - p$.

This implication can be expressed in the following way:

(9) If it is necessary that p or that $\text{non} - p$, it is possible that p and that $\text{non} - p$.

Then (9) is equivalent to (5), and we obtained a sound result:

$$L(p \text{ or } \sim p) \text{ is equivalent to } M(p \text{ and } \sim p).$$

This rebuilding of the argument shows us that the absurd obtained by Aristotle and Lukasiewicz does not subsist, if we make a correct representation of the modal proposition or statement. The absurd does not subsist because the modal functions (necessary and non-necessary), which appear in both sides of the statement obtained by (1) and (2) are linked to different connectors or functors, in one side *or* (disjunction) and in the other side *and* (conjunction).

Thus, the argument is sound, if it conserves the necessary articulation between functors and modalities, other great discovery of Aristotle.

TEMPUS REGIT MODUM

On the other side, assuming the necessary articulation between functors and modalities, we must identify what is the great frame of time which governs the

⁹We must agree that this is not the place to discuss the interesting question: what is really a proposition?

modality. This is another very important discovery of Aristotle's: *tempus regit modum*. Therefore, concerning the past, we can say:

It is necessary¹⁰ that p or that $non - p$, but in this case the sentence "it is necessary that p or that $non - p$ " hides in fact two sentences:

(i) "it is necessary that p and (ii) it is necessary that $non - p$."

The reason is the different ontological status: one from the two sentences should be true, the other should be false. Here, we can write, the following formulae, as the existence of the past guarantees the distributivity of the necessity, :

$$L(p \text{ or } non - p) \equiv Lp \text{ or } L\sim p \Rightarrow \sim M(p \text{ or } non - p) \equiv \sim Mp \text{ or } \sim M\sim p.$$

$M(p \text{ and } \sim p)$ is not sound here, because *possible* is a modality exclusive for the future. Thus we can admit $\sim M$ and its consequences, therefore:

$$\begin{aligned} Lp &\equiv \sim M\sim p, \\ L\sim p &\equiv \sim Mp, \\ \sim\sim M\sim p &\equiv \sim Mp, \\ \sim\sim Mp &\equiv \sim M\sim p. \end{aligned}$$

The Rebuilding of the Argument and its Consequences

One of the objectives of the earlier Polish Logic school was to reinterpret the philosophical problems according to principles of contemporaneous logic. In the case of the rebuilding that I exposed, it now seems evident that most likely the chapters XII and XIII of *De Interpretatione* were written a long time after chapter IX was produced. This fact can explain that, in rewriting the book, Aristotle wrote schematic formulae which could not express the results obtained in chapter IX. As a consequence of these formulae appeared the error in the Aristotle's argument that we discussed above. Lukasiewicz repeated this error in formal modal language, in his representation of modal propositions in Aristotle. This error is not only an error of logical inference, but it is more an error of the symbolic representation of the modal sentences in Aristotle's modal logic. In fact, the possibility, if it concerns *stricto sensu* the future, should be represented by the following sentence:

"It is possible that p and that $non - p$ ", because p and $non - p$ have the same ontological status. Here we can not separate " p " from " $non - p$ ", both are parts of the same dictum. Dictum and modality form only one sentence.

By the same reason, concerning the future, we write: "It is necessary that p or that $non - p$ ", as one inseparable sentence." In order to use the Kant's

¹⁰I would prefer capital letters in the place of the lowercases ones, because the dictum looks like more a predicate than a proposition.

terminology, we can say that p and $\sim p$ are the matter of the same judgment and the relations between modality and p and $\sim p$ give us its form.¹¹

A formal representation of modalities, where there is crossing between possibility and necessity, in Aristotle's modal logic, should be exhibit as one of its axioms, if we want to be faithful to chapter IX of *De Interpretatione*. We have the following equivalence:

$L(p \text{ or } non - p) \equiv M(p \text{ and } non - p)$, where $L(p \text{ or } non - p)$ does not imply Lp or $L\sim p$, and $M(p \text{ and } \sim p)$ does not imply Mp and $M\sim p$.

The critical examination of the error in Lukasiewicz and Aristotle's arguments allows us to highlight very important aspects of a proposition and it helps us to understand what makes a proposition *one* proposition. For example, why $L(p \text{ or } non - p)$ means two sentences, if this formula concerns the past, and only one sentence, if this formula concerns the future.

Our reconstruction showed us that Aristotle's logic has not only historical interest, but it remains very fertile, despite the dominant opinion.

¹¹ "In jedem Urteile kann man die gegebenen Begriffe logische Materie (zum Urteile), das Verhältniss derselben (vermittels der Kopula) die Form des Urteils nennen." Kritik der reinen Vernunft, Materie und Form, Elementarlehre II, T.I. Abt. Buch Anhang, Philip Reclam, 2010, p. 352.